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The diophantine equation $2^n = x^2 + 7$

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This paper deals with the following

Theorem. *The only solutions in integers $x > 0$ of the equation*

$$2^n = x^2 + 7 \tag{1}$$

are given by

$$\begin{aligned} &n = 3,\ x = 1,\\ &n = 4,\ x = 3,\\ &n = 5,\ x = 5,\\ &n = 7,\ x = 11,\\ &n = 15,\ x = 181. \end{aligned} \tag{2}$$

In 1913, Ramanujan gave these values (2) in Problem (465), page 120 of Vol. 5 of the *Journal of the Indian Mathematical Society*, and asked whether there were other values of n. In Ramanujan's collected works, there is a reference on page 327 to "Solution by K. J. Sanjana and T. P. Trevedi on pages 227, 228 also of Vol. 5." This, however, is merely a verification for some values of n.

On page 272 of Nagell's *Introduction to Number Theory*, the theorem is set as a problem. The enunciation is preceded by the problem, to show by considering the quadratic field $R(\sqrt{-7})$ in which factorization is unique, that the only rational integer solutions of

$$x^2 + x + 2 = y^3 \tag{3}$$

are given by $y = 2$. It seems to be implied that the same method will suffice for a proof of the theorem.

The theorem was proved by Chowla, D. J. Lewis, and Skolem in a joint paper submitted in 1958 for publication in the *Proceedings of the American Mathematical Society*.[1] The question was brought to my notice by Professor Chowla. I have found the present solution which is entirely different from theirs, which I had not seen when this paper was written.

[1] It has since appeared in Vol. 10 (1959) 663–669. Professor Nagell now informs me that he published (in Norwegian) a simple proof of the theorem in the Norsk Matematisk Tidsskrift 30 (1948) 62–64.