Communicated 9 Oktober 1963 by LENNART CARLESON

On the boundary values of harmonic functions in R^3

By KJELL-OVE WIDMAN

1. Introduction

The purpose of this paper is to exhibit three theorems about the boundary values of harmonic functions, defined in R^3 regions which are bounded by Liapunov surfaces. Theorem 1 shows the existence almost everywhere of non-tangential boundary values of positive harmonic functions. A full proof of this theorem is given. Theorem 2 assures the existence almost everywhere of non-tangential boundary values for functions bounded in cones with vertex on the boundary and lying in the region. Theorem 3, finally, gives a necessary and sufficient condition for the existence of non-tangential boundary values, originally derived by Marcinkiewicz and Zygmund and later generalized by Stein. As the proofs of the two latter theorems differ from proofs published elsewhere only in the technical aspect, these are not included here.

2. Definitions

We consider an open region Ω_1, bounded by a Liapunov surface S_1. By Liapunov surface we mean a closed, bounded surface with the following properties:

1°. At every point of S_1 there exists a uniquely defined tangent plane, and thus also a normal.

2°. There exist two constants $C'>0$ and λ, $0<\lambda\leqslant 1$, such that if θ is the angle between two normals, and r is the distance between their foot points, the following inequality holds $\theta < C' \cdot r^{\lambda}$.

3°. There exists a constant $d>0$ such that if Σ is a sphere with radius d and center Q_0 on the surface, a line parallel to the normal at Q_0 meets S_1 at most once inside Σ. It is easily realized that d may be chosen arbitrarily small.

For the properties of Liapunov surfaces see Gunther [5]. In the sequel we shall consider only inner normals, which will simply be referred to as normals. We denote by $V(Q, \alpha, h)$ a right circular cone having vertex at $Q \in S_1$, axis along the normal at Q, altitude h, generating angle $=\alpha$ and being contained in Ω_1. Non-tangential approach to the boundary means approach inside some $V(Q, \alpha, h)$. $r(P, Q)$ will be the distance between $P \in \Omega_1$ and the tangent plane at $Q \in S_1$. The volume element in R^3 will be denoted by dv and the surface element by dS.