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On a diophantine equation in two unknowns

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§ 1.

The purpose of this paper is to examine the solvability in integers x and y of the equation

$$x^2 + x + \tfrac{1}{4}(D + 1) = y^q \tag{1}$$

where D is a positive integer $\equiv 3 \pmod{4}$ and q denotes an odd prime.

The special case $D = 3$ has already been treated by T. NAGELL, who showed that the equation[1]

$$x^2 + x + 1 = y^n$$

is impossible in integers x and y, when $y \neq \pm 1$, for all whole exponents $n\,(> 2)$ not being a power of 3.

W. LJUNGGREN completed this result by proving that the equation[2]

$$x^2 + x + 1 = y^3$$

has the only solutions $y = 1$ and $y = 7$. Thus it is sufficient in (1) to take $D \geqq 7$. *We furthermore suppose that D has no squared factor* > 1.

According to a theorem of AXEL THUE the equation (1) has only a finite number of solutions in integers x and y, when D and q are given.[3]

§ 2.

If we put $\varrho = \frac{1}{2}(-1 + \sqrt{-D})$ and $\varrho' = \frac{1}{2}(-1 - \sqrt{-D})$, the equation (1) can be written

$$(x - \varrho)(x - \varrho') = y^q. \tag{1'}$$

ϱ and ϱ' are conjugate integers in the quadratic field $K(\sqrt{-D})$. The numbers 1, ϱ form a basis of the field.

The two principal ideals

$$(x - \varrho) \quad \text{and} \quad (x - \varrho')$$

are relatively prime. To show it we denote by $\mathfrak{j}$ their highest common divisor. The number $2x + 1 = 2x - (\varrho + \varrho')$ is contained in $\mathfrak{j}$ and also the number $D = -(\varrho - \varrho')^2$. If we write the equation (1) in the form