LOCALIZATION AND SUMMABILITY OF MULTIPLE FOURIER SERIES

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Introduction

1. Definitions

In this paper we shall deal with the theory of "spherical" summability of multiple Fourier series.

Let $f(x) = f(x_1, x_2 \ldots x_k)$ be a Lebesgue integrable function defined on the fundamental cube Q_k, $-\pi < x_i \leqslant \pi$, $i = 1, \ldots k$, in Euclidean k-space. We form the Fourier series of $f(x)$

$$f(x) = \sum a_n e^{in\cdot x} = \sum a_{n_1 n_2 \cdots n_k} e^{i(n_1 x_1 \cdots + n_k x_k)}, \tag{1.1}$$

where $n = (n_1, \ldots, n_k)$ is a vector with integral components, $n \cdot x = n_1 x_1 + n_2 x_2 \cdots + n_k x_k$, with

$$a_n = (2\pi)^{-k} \int_{Q_k} f(x)\, e^{-in\cdot x}\, dx,$$

and $dx = dx_1\, dx_2 \ldots dx_k$.

We next form the spherical Riesz means of order δ of $f(x)$

$$S_R^\delta(x) = S_R^\delta(x, f) = \sum_{|n| < R} \left(1 - \frac{|n|^2}{R^2}\right)^\delta a_n e^{in\cdot x}, \tag{1.2}$$

where $|n| = (n_1^2 + \cdots + n_k^2)^{\frac{1}{2}}$. Unless stated to the contrary, we shall assume that $k \geqslant 2$.

The general problem of the theory concerns itself with the validity (and meaning) of

$$\lim_{R\to\infty} S_R^\delta(x, f) = f(x), \tag{1.3}$$

for some appropriate δ.

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