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Extension of smooth CR mappings between non-essentially finite hypersurfaces in $\mathbf{C}^3$

Henri-Michel Maire and Francine Meylan(1)

0. Introduction

Let M be a real analytic hypersurface in $\mathbf{C}^3$ containing 0 and let M' be the algebraic hypersurface in $\mathbf{C}^3$ defined by

$$\operatorname{Im} w' = |z_1'|^2 + \operatorname{Re} w' |z_2'|^2, \quad (z_1', z_2', w') \in \mathbf{C}^3. \tag{0.1}$$

For any $b'<0$, the function $(z', w') \mapsto 1/(w'-ib')$ is holomorphic in $\mathbf{C}^3 \setminus \{w'=ib'\} \supseteq M'$; therefore its restriction to M' is a CR function which does not extend holomorphically around $(0,0,ib')$. A classical argument using Baire's category theorem (see [HT, p. 125]) guarantees the existence of a CR function on M' which does not extend to a full neighborhood of $0 \in \mathbf{C}^3$. In contrast, for CR mappings we have the following result.

Theorem 1. *If $h: M \to M'$ is a smooth CR local diffeomorphism at* 0 *with $h(0)=0$, then h extends to a holomorphic mapping in a full neighborhood of* 0 *in $\mathbf{C}^3$.*

As we shall see in Corollary 1.2, if h satisfies the hypothesis of the theorem (more generally if h is of finite multiplicity) then M is of finite type. After Trepreau's theorem we know that any CR function on M extends holomorphically to *one* side of M; therefore Theorem 1 is equivalent to a reflection principle (cf. Baouendi and Rothschild [BR3]). Because we do not assume M to be algebraic, and because M' is not essentially finite, Theorem 1 does not follow from the recent results of Baouendi, Huang and Rothschild [BHR] nor from those of Baouendi and Rothschild [BR2]. Notice that M' is holomorphically non-degenerate in the sense of Stanton [S].

Theorem 1 may be generalized as follows.

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