

# ON THE STEADY-STATE SOLUTIONS OF THE NAVIER-STOKES EQUATIONS, III

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## Contents

	Page
Introduction . . . . .	197
. Notation and definitions; preliminary estimates; the representation formula . . . .	201
2. A priori estimation of the Dirichlet Integral . . . . .	205
2 a) Estimation of the Dirichlet Integral in a bounded region . . . . .	208
2 b) Estimation of the Dirichlet Integral in an exterior region; case of zero outflux .	211
2 c) Estimation of the Dirichlet Integral in an exterior region; general case . . . .	213
3. A priori estimation of the solution . . . . .	215
3 a) Estimation of the solution in a bounded region . . . . .	218
3 b) Estimation of the solution in an exterior region . . . . .	219
4. Behaviour at infinity; the representation formula . . . . .	223
5. Existence theorems . . . . .	226
5 a) Existence of a solution in a finite region . . . . .	226
5 b) Existence of a solution in an exterior region . . . . .	227
6. Remarks on the preceding sections; an example . . . . .	228
7. Transition to zero Reynolds' Number . . . . .	231
7 a) Transition to zero Reynold' Number; case of a bounded region . . . . .	231
7 b) Transition to zero Reynolds' Number; exterior region . . . . .	234
7 c) Transition of the force exerted on a fluid interface . . . . .	240
8. Uniqueness and continuous dependence . . . . .	241

## Introduction

In this work we study the relations connecting a solution of the Navier-Stokes equations

$$\begin{aligned}\mu \Delta \mathbf{w} - \varrho \mathbf{w} \cdot \nabla \mathbf{w} - \nabla p &= 0 \\ \nabla \cdot \mathbf{w} &= 0,\end{aligned}\tag{1}$$

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