CHARACTERISTIC VALUES ASSOCIATED WITH A CLASS OF NONLINEAR SECOND-ORDER DIFFERENTIAL EQUATIONS

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1. The equations to be studied in this paper are of the form

$$y'' + y\,F(y^2, x) = 0, \tag{1.1}$$

or, more generally,

$$y'' + p(x)\,y + y\,F(y^2, x) = 0, \tag{1.2}$$

where $p(x)$ is a positive and continuous function of x in a finite closed interval $[a, b]$, and the function $F(t, x)$ is subject to the following conditions:

$F(t, x)$ is continuous in t and x for $0 \leqslant t < \infty$ and $a \leqslant x \leqslant b$, respectively; (1.3 a)

$F(t, x) > 0$ for $t > 0$ and $x \in [a, b]$; (1.3 b)

There exists a positive number ε such that, for any x in $[a, b]$, $t^{-\varepsilon} F(t, x)$ is a non-decreasing function of t for $t \in [0, \infty]$. (1.3 c)

The statement that a function $y(x)$ is a solution of (1.1) or (1.2) in an interval $[a, b]$ will mean that $y(x)$ and $y'(x)$ are continuous in $[a, b]$ and that $y(x)$ satisfies there the equation in question.

Because of condition (1.3 c), equation (1.2) is not included in the class (1.1) and must be considered separately. Condition (1.3 b) and, in the case of equation (1.2), the fact that $p(x) > 0$ shows that a solution $y(x)$ of (1.1) or (1.2) satisfies the inequality $y y'' < 0$ for $y \neq 0$, i.e., the solution curves are concave with respect to the horizontal axis. It follows therefore from an elementary geometric argument that any solution $y(x)$ for which y and y' are finite at some point of $[a, b]$, can be continued to all points of the interval.

Our aim is to investigate the properties of those solutions $y(x)$ of (1.1) or (1.2) which satisfy the boundary conditions $y(a) = y(b) = 0$, although most of our results