Ark. Mat., **34** (1996), 265–283

Counting eigenvalues using coherent states with an application to Dirac and Schrödinger operators in the semi-classical limit

William Desmond Evans, Roger T. Lewis, Heinz Siedentop and Jan Philip Solovej

1. Introduction

Coherent states have been successfully used for obtaining leading order asymptotics for spectral properties of Schrödinger operators (e.g., Berezin [2], Lieb [11], and Thirring [16]) and other systems. They have been used so far mainly to *sum* the negative eigenvalues, i.e., computing the first Riesz moment, or to evaluate traces of other convex or concave functions of the operator using the Berezin–Lieb inequalities. Another quantity of interest is the dimension of the discrete spectral subspace. At this point we would like to mention the paper of Li and Yau [10] which actually does not mention coherent states at all but which may be reinterpreted in terms of coherent states. Li and Yau treat the counting problem for Schrödinger operators H in a compact domain and evaluate the trace of the semi-group which requires H to have discrete spectrum only. In some sense our approach is related to theirs.

The purpose of this note is to show that the counting problem is accessible to a coherent states analysis: based on some rudimentary functional calculus we roll the problem back to the first Riesz moment. However, instead of developing a general theory we demonstrate the usefulness of the technique with a nontrivial example, the Dirac operator in the semi-classical limit, but our general result also includes the Schrödinger operator. We shall recover the leading order asymptotics of the number of eigenvalues in the spectral gap of the Dirac operator, namely—as predicted by Planck—the phase space volume of the corresponding energy shell divided by the Planck constant h raised to the power d where d is the underlying dimension, which in this case is three. In the following we shall choose the units such that the spectrum of the free Dirac operator is $(-\infty,-1]\cup[1,\infty)$, namely, in physical terms, such that the velocity of light and the rest energy of the electron