A THEOREM OF COMPLETENESS FOR COMPLEX ANALYTIC FIBRE SPACES

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1. Introduction

We begin by recalling several definitions, introduced in the authors' paper [3], concerning complex analytic families of complex manifolds.

By a complex analytic fibre space we mean a triple $(\mathcal{V}, \varpi, M)$ of connected complex manifolds $\mathcal{V}$, M and a holomorphic map ϖ of $\mathcal{V}$ onto M. A fibre $\varpi^{-1}(t)$, $t \in M$, of the fibre space is *singular* if there exists a point $p \in \varpi^{-1}(t)$ such that the rank of the jacobian matrix of the map ϖ at p is less than the dimension of M.

DEFINITION 1. We say that $\mathcal{V} \xrightarrow{\varpi} M$ is a complex analytic family of compact, complex manifolds if $(\mathcal{V}, \varpi, M)$ is a complex analytic fibre space without singular fibres whose fibres are connected, compact manifolds and whose base space M is connected.

With reference to a complex manifold $V_0 = \varpi^{-1}(0)$, $0 \in M$, we call any $V_t = \varpi^{-1}(t)$, $t \in M$, a deformation of V_0 and we call $\mathcal{V} \xrightarrow{\varpi} M$ a complex analytic family of deformations of V_0.

DEFINITION 2. A complex analytic family $\mathcal{V} \xrightarrow{\varpi} M$ of compact, complex manifolds is (complex analytically) complete at the point $t \in M$ if, for any complex analytic family $\mathcal{W} \xrightarrow{\pi} N$ such that $\pi^{-1}(0) = \varpi^{-1}(t)$ for a point $0 \in N$, there exist a holomorphic map $s \rightarrow t(s)$, $t(0) = t$, of a neighborhood U of 0 on N and a holomorphic map g of $\pi^{-1}(U)$ into $\mathcal{V}$ which maps each fibre $\pi^{-1}(s)$, $s \in U$ of $\mathcal{W}$ biregularly onto $\varpi^{-1}(t(s))$. The complex analytic family $\mathcal{V} \xrightarrow{\varpi} M$ is called (complex analytically) complete if it is (complex analytically) complete at each point t of M.