GLOBAL BOUNDARY BEHAVIOR OF MEROMORPHIC FUNCTIONS

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1. Introduction

Let f be a function sending the open unit disk D into the Riemann sphere S. A point y on S is in the *global cluster set* of f, denoted by $C(f)$, if and only if there exists a sequence of points z_n in D such that $\lim |z_n| = 1$ and $\lim f(z_n) = y$. Thus, for example, if f is continuous on D, and can be extended to be continuous on $\bar{D}$, then $C(f)$ is the image of the bounding circle and hence a Peano space.

If f is continuous, then $C(f)$ is a continuum. Conversely, it is easy to prove that any continuum C on the sphere S is the global cluster set for some continuous function f. Collingwood ([3], p. 123) and Cartwright asked whether every continuum on S is the global cluster set of a function f meromorphic on the open disk D. D. B. Potyagailo [8] and W. Rudin [10] independently gave as counter-example the continuum consisting of the union of (a) a spiral, $r = \theta/(\pi + \theta)$, $\pi \leqslant \theta < \infty$, (b) the unit circumference, and (c) an interval, $1 \leqslant x \leqslant 2$, $y = 0$.

Because this example is not locally connected, and because, if f is continous on $\bar{D}$, then $C(f)$ is locally connected, one might conjecture that every locally connected continuum is the global cluster set for some function f meromorphic on D. In Section 2, we give a counter-example to this conjecture. The example also answers in the negative a question of Gerald MacLane [5]: Is every Peano space the image of the bounding circle of a function f meromorphic on D and continuous on $\bar{D}$?

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