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Geometric localization, uniformly John property and separated semihyperbolic dynamics

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0. Introduction

Different types of geometric localization are used extensively in analysis. Localization grasps fine properties of the boundary which allows one to carry out estimates of harmonic measure, Green's function, etc. In the absence of the Riemann mapping theorem, localization may serve as a weak substitute. Examples of such an approach can be found in Ancona [A1], [A2], J.-M. Wu [W] for (mainly) Lipschitz domains and in Jones [J1], Jerison and Kenig [JK] for non-tangentially accessible domains. Carleson's work [C] may be considered as a source for this approach. In this paper we are going to deal with a localization property for John domains. Our motivation is the following. Denote by $A_\infty(f)$ the domain of attraction to ∞ of a polynomial f. A recent result of L. Carleson, P. W. Jones and J. C. Yoccoz ([CJY]) shows that $A_\infty(f)$ is a John domain if and only if f is semihyperbolic. In the first section we prove localization for simply connected John domains. In Section 2 we give an example showing that localization fails for arbitrary John domains and we prove the localization at a fixed scale. The third section is devoted to some geometric properties of the Julia set of a semihyperbolic polynomial. In the fourth section we introduce separated semihyperbolic dynamics and prove that the localization of $A_\infty(f)$ is equivalent to the property of being separated semihyperbolic. For example localization works for critically finite f. In Section 5 we show that localizability is equivalent to uniformity for John domains and Section 6 provides an example of semihyperbolic but not separated semihyperbolic polynomials. In the last section, we discuss some applications of this property. Commenting on Sections 1–3 let us mention that throughout them we modify ideas virtually present in [BH], [CJY],

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