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A general discrepancy theorem

H.-P. Blatt and H. N. Mhaskar

Dedicated to Professor Meinardus on the occasion of his sixtyfifth birthday.

1. Introduction

Let E be a Jordan curve or a Jordan arc and let σ be a signed measure on E. We define the *discrepancy* of σ by

$$D[\sigma] := \sup |\sigma(J)| \tag{1.1}$$

where the supremum is taken over all subarcs $J \subseteq E$. If $\{\nu_n\}$ is a sequence of Borel measures on E converging to a Borel measure μ in the sense that $D[\nu_n - \mu] \to 0$ as $n \to \infty$, then $\{\nu_n\}$ converges to μ in the weak-star sense. Thus, the discrepancy *between* ν_n *and* μ defined by $D[\nu_n - \mu]$ serves as a measurement on the rate of the weak-star convergence. Therefore, many mathematicians ([1]–[4], [7], [8], [10], [12], [13], [15], [16], [18], [22], [30], [32], [34]) have studied the notion of discrepancy of a signed measure under various conditions. Often, the discrepancy is estimated in terms of the logarithmic potential $U(\sigma, z)$ defined for any signed measure σ by

$$U(\sigma, z) := \int \log \frac{1}{|z-t|}\, d\sigma(t). \tag{1.2}$$

A typical result is the following estimate due to Ganelius.

Theorem 1.1 ([13]). *Let ν be a positive unit Borel measure on the unit circle and $d\mu := dt/2\pi$. Then*

$$D[\nu - \mu] \le c \Big| \inf_{|z|=1} U(\nu - \mu, z) \Big|^{1/2}, \tag{1.3}$$

where c is an absolute constant independent of ν.

For example, consider a monic polynomial p_n of degree n having all of its zeros on the unit circle and let ν_n be the unit measure associating the mass $1/n$ with