COMPLETELY INTEGRABLE DIFFERENTIAL EQUATIONS IN ABSTRACT SPACES.[1]

BY

A. D. MICHAL and V. ELCONIN

of PASADENA, CALIFORNIA.

Introduction. The primary object of this paper is to obtain existence theorems for the abstract completely integrable[2] differential equation

$$d_\xi^x f(x) = F(x, f(x), \xi),$$

where the left member is the Fréchet differential[3] of $f(x)$ with increment ξ, and the ranges and domains of the functions involved are in Banach spaces.[4] By a generalization of the well-known method of successive approximations and the use of most of the known and several new properties of abstract differentials and integrals, we prove two main theorems, one local in character and the other 'in the large', by means of which we obtain new existence theorems for Pfaffian differential equations in Hilbert space and the well-known space of continuous functions, and also a new existence theorem for abstract implicit functions. Kerner's recent theorem[5], in which $F(x, f(x), \xi)$ is independent of $f(x)$, is an immediate corollary of the first main theorem. By specializing the Banach spaces, we obtain several of the recent improvements in the theory of the classical

[1] Presented to the Amer. Math. Soc. (1934). Cf. Bull. Amer. Math. Soc., 40, 530 (1934).

[2] The condition of complete integrability (the premise in (ii) of Theorem I) is suggested by a theorem of Kerner on the symmetry in the increments of a repeated Fréchet differential; it is definitely a necessary condition for the existence of the solution in Theorem II.

[3] M. FRÉCHET, Annales Sc. Ec. Norm. Sup., t. 42, 293—323 (1925). See also T. H. HILDEBRANT and L. M. GRAVES, Trans. Amer. Math. Soc., 29 (1927).

[4] S. BANACH, Fund. Math., 3, 133—181 (1922). See also his book, Théorie des opérations linéaires, (1932). A Banach space is briefly a complete normed vector space closed under multiplication by real numbers.

[5] M. KERNER, Annals of Math. (1933).