

PERTURBATIONS OF NONLINEAR SYSTEMS

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1. Introduction

In section 3 of this paper we extend the author's results [4] concerning perturbations of real linear systems of the form

$$\dot{x} = [A(t) + B(t)]x, \quad (1.1)$$

where $A(t)$ and $B(t)$ are continuous and uniformly bounded matrices. In particular we obtain conditions on the matrix $A(t)$ which will assure us that the characteristic exponents of system (1.1) are continuous at $B(t) \equiv 0$ as functions of $B(t)$ (Corollary 3.1). In section 4 we obtain results concerning the existence and variation of bounded solutions of nonlinear differential equations (Theorem 4.1). In section 5 we apply these results to almost periodic nonlinear systems and extend the author's previous results (Theorem 5.1).

2. Elementary Transformations and Definitions

A counter example due to Perron [7] shows that even in the case where the matrix $A(t)$ in (1.1) is a diagonal matrix the characteristic exponents of (1.1) need not be continuous at $B(t) \equiv 0$. One notes, however, that one of the diagonal terms in Perron's example fails to possess a mean value. We shall find that if one restricts oneself to matrices $A(t)$ which are kinematically similar [5] to upper triangular matrices whose diagonal elements have the following property I, then the characteristic exponents of system (1.1) are continuous at $B(t) \equiv 0$.

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