

PREDICTION THEORY AND FOURIER SERIES IN SEVERAL VARIABLES. II

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1. Introduction

This second paper on Prediction Theory, like our first one [10], is divided into two parts: the first, consisting of the first eight sections, treats complex-valued functions defined on rather general groups, and the second part deals with matrix-valued functions defined on the unit circle. In both parts we are concerned with degeneracies which were excluded by our hypotheses before, but which turn out to be interesting from both the function-theoretic and the prediction-theoretic points of view.

Unlike the first paper, this one has to do with difficulties which do not exist at all for the classical case of scalar functions defined on the circle group. Both parts of the paper leave interesting problems unsolved. In this introduction we shall try to present the questions of this paper and our contribution to their solution in broad terms.

Let $f = f(e^{ix})$ be a summable function defined on the circle whose Fourier series has the form

$$f(e^{ix}) \sim \sum_{n \geq 0} a_n e^{nix}.$$

It is important and well-known that

$$\int \log |f| d\sigma^{(2)}$$

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⁽²⁾ $d\sigma$ always denotes normalized invariant measure on the compact group being considered. Here it is $dx/2\pi$ on $(0, 2\pi)$.