

SPECTRAL MEASURES IN LOCALLY CONVEX ALGEBRAS

BY

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Introduction

The central subject of classical spectral theory has been the spectral representation of self-adjoint and normal operators in Hilbert space; after Hilbert had given a complete treatment of the bounded case, the spectral theory of unbounded self-adjoint and normal operators was developed by von Neumann and others. (An excellent account can be found in [23].) Abstractions of algebras of Hermitian and normal operators were considered by various authors, notably Stone [22] who characterized such algebras as algebras of continuous functions on a compact (Hausdorff) space. Investigations by Freudenthal [9] and Nakano [15] (especially papers 1 and 2), also leading to spectral theories, went in a different direction. Generalizations of unitary operators to reflexive Banach spaces were considered by Lorch [13]; Lorch also developed an operational calculus for those operators in reflexive Banach spaces that can be represented by a spectral measure [12]. (Taylor [24] developed such a calculus for closed operators on a Banach space whose spectrum does not cover the plane, but necessarily restricted to functions locally holomorphic on the spectrum. There are some recent results in this direction for operators on a locally convex space [26], [16].) The most extensive work on bounded and unbounded operators in a Banach space is due to Dunford, Schwartz, Bade and others (for a detailed bibliography, see [6] and [8]). Dunford considers operators that have a countably additive resolution of the identity, but may differ from a spectral operator (in the sense of Definition 4, Section 4 of this paper) by a quasi-nilpotent. A survey of this work is given in [6]. Other contributions were made by Bishop [3], and most recently results on spectral operators (in Dunford's sense) in locally convex spaces were announced by Tulcea [25].

Since the time when the spectral theory of bounded and unbounded normal operators in Hilbert space took definite shape, the theory of topological, in particular, of locally

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