ON THE STRUCTURE OF MEASURE SPACES

BY

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1. Introduction

In two papers fundamental to the theory of measure, Halmos and von Neumann [3] and Maharam [4] have characterized the measure algebras associated with totally finite measure spaces by showing that each such algebra is isomorphic to the measure algebra of some canonical measure space. In this note, we shall show that the measurable sets of a given measure space are constructed of certain null subsets of that space in essentially the same manner as the measurable sets of the standard measure space, to which the given one is isomorphic, are constructed of points. The technique used depends on a theorem concerning the relation between the lattices of measurable functions modulo null functions defined on isomorphic measure spaces. We conclude the discussion with some applications to problems that arise in connection with the study of a theorem of Saks and Sierpiński [5] on the approximation of real functions by measurable functions.

2. Preliminary considerations

Let $(X, \mathbf{S}, \mu)$ be a measure space, and let $\mathbf{N}$ be the class of all measurable sets of measure zero. If E and F are elements of $\mathbf{S}$, and if $\triangle$ denotes the operation of symmetric difference, we write $E \sim F$ if and only if $E \triangle F$ belongs to $\mathbf{N}$. The relation $\sim$ thus defined on $\mathbf{S}$ is an equivalence relation. The quotient space $\mathbf{S}/\mathbf{N}$ to which $\sim$ gives rise, is denoted by $\mathbf{S}(\mu)$. If E is an element of $\mathbf{S}$, we denote by $[E]$ the equivalence class determined by E. The binary operations $+$, $\cdot$, are (well) defined on $\mathbf{S}(\mu)$ by means of the equations

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