ON SOLUBLE IRREDUCIBLE GROUPS OF LINEAR SUBSTITUTIONS IN A PRIME NUMBER OF VARIABLES

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It is well known that if a transitive permutation group of prime degree is soluble it must be cyclical or metacyclical; so that if the degree be p, the order of the group is pr, where r is equal to or is a factor of $p-1$.

I propose here to consider the corresponding question for an irreducible group of linear substitutions in a prime number of variables; and in particular to determine the numbers which may be the order of such a group when it is soluble.

1. A group of linear substitutions in p variables is called irreducible when it is impossible to find $q(<p)$ linear functions of the variables which are transformed among themselves by every operation of the group. It has recently been shown by Herr FROBENIUS[1] that if a group G, of finite order, is isomorphic (simply or multiply) with an irreducible group of linear substitutions in p variables, then p must be a factor of the order of G.

A group of linear substitutions in p symbols, which is of finite order and ABELIAN, is always completely reducible[2]; *i. e.*, a set of p independent

[1] Berliner Sitzungsberichte, 1896, p. 1382.

[2] I am not aware whether a separate proof of this statement has been published; but it is contained as a special case in Herr FROBENIUS's investigations in the *Berliner Sitzungsberichte* on the representation of a group by means of linear substitutions.