REDUCIBILITY OF GENERALIZED PRINCIPAL SERIES REPRESENTATIONS

BY

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1. Introduction

Let G be a connected semisimple matrix group, and $P \subseteq G$ a cuspidal parabolic subgroup. Fix a Langlands decomposition

$$P = MAN$$

of P, with N the unipotent radical and A a vector group. Let δ be a discrete series representation of M, and ν a (non-unitary) character of A. We call the induced representation

$$\pi(P, \delta \otimes \nu) = \mathrm{Ind}_P^G (\delta \otimes \nu \otimes 1)$$

(normalized induction) a *generalized principal series representation*. When ν is unitary, these are the representations occurring in Harish-Chandra's Plancherel formula for G; and for general ν they may be expected to play something of the same role in harmonic analysis on G as complex characters do in $\mathbf{R}^n$. Langlands has shown that any irreducible admissible representation of G can be realized canonically as a subquotient of a generalized principal series representation (Theorem 2.9 below). For these reasons and others (some of which will be discussed below) one would like to understand the reducibility of these representations, and it is this question which motivates the results of this paper. We prove

THEOREM 1.1. (Theorems 6.15 and 6.19). *Let $\pi(P, \delta \otimes \nu)$ be a generalized principal series representation. Fix a compact Cartan subgroup T^+ of M (which exists because M has a discrete series). Let*

$$\mathfrak{h} = \mathfrak{t}^+ + \mathfrak{a}, \mathfrak{g}$$

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