THE LOCAL REAL ANALYTICITY OF SOLUTIONS TO $\square_b$ AND THE $\bar{\partial}$-NEUMANN PROBLEM

BY

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Since the introduction of the celebrated $\bar{\partial}$-Neumann problem by D. C. Spencer in the 1950's, with related problems being studied by Garabedian and Spencer [12], Kohn and Spencer [21], and Conner [5], regularity properties of solutions of non-elliptic partial differential equations have been closely linked with important questions in the field of several complex variables, as well as purely real variable questions such as the real analytic embedding theorem of Morrey [23]. The C^∞ regularity results of Kohn [18], later simplified in [20], gave rise to much interest in higher regularity properties both for the $\bar{\partial}$-Neumann problem and, at the same time, interior problems for operators such as the "complex boundary Laplacian", $\square_b$, which appear elliptic in most directions and suffer a loss of one derivative overall, whence the name "subelliptic".

In the elliptic case, either for interior or coercive boundary value problems, local regularity of solutions has been well established for some time [24, 25] in the C^∞, Gevrey, and real analytic categories. The proofs are of two kinds: "classical" proofs, which rely exclusively on L^2 methods (Gårding's inequality) plus carefully chosen localizing functions, and proofs employing pseudo-differential operators as introduced by Friedrichs and Hörmander. This theory has been widely developed in recent years and broadened to treat non-elliptic problems, either by means of Fourier Integral Operators or by hyperfunction techniques, and special classes of such operators have been introduced to analyze the behavior of operators such as $\square_b$ [9, 4], guided in large measure by results and methods in the theory of nilpotent Lie groups such as the Heisenberg group [11]. And the conviction, on the part of many, that $\square_b$ should be locally analytic hypoelliptic was based, perhaps, at first largely on the explicit fundamental solution for this operator on the Heisenberg group which is analytic off its singularity, cf. [11].

On the other hand, initial efforts to prove that these operators enjoyed high regularity