SOME RIGIDITY THEOREMS FOR MINIMAL SUBMANIFOLDS OF THE SPHERE

BY

D. FISCHER-COLBRIE (1)

Columbia University, New York, N.Y., U.S.A.

Introduction

It is well known that the regularity of minimal submanifolds can be reduced to the study of minimal cones and hence to compact minimal submanifolds of the sphere. A phenomenon related to regularity was discovered by Bernstein [3] in 1915. The Bernstein Theorem says that an entire solution to the minimal surface equation in two variables is a plane. An answer to the extrinsic rigidity question of whether a minimal submanifold which lies in some neighborhood of a standard sphere must actually be a standard sphere is of interest in relation to the above topics as well as in its own right.

Efforts to generalize Bernstein's Theorem were made by many authors. The work of Simons [16] completed the proof in codimension one up to dimension 7. Bombieri, de Giorgi and Guisti [4] gave a counterexample in dimension 8. For two-dimensional graphs, Osserman [14] proved a version of Bernstein's theorem assuming the normal vectors omit a neighborhood of the sphere. Simons [16] proved that a minimal cone whose normal planes lie in a sufficiently small neighborhood is a plane and hence Bernstein's theorem is true for a graph whose normals satisfy the same condition. Reilly [15] enlarged the neighborhood. His estimate says that if a cone has the property that the normals satisfy $\langle N, A\rangle > \sqrt{(2k-2)/(3k-2)}$ for some fixed k-plane A, then the cone is a plane. For two-dimensional minimal graphs Barbosa [2] improved the neighborhoods to an open hemisphere. More specifically, Barbosa showed that a compact minimally immersed sphere in S^{k+2} such that its normal satisfy $\langle N, A\rangle > 0$ for some fixed A is totally geodesic. The theorem was also proved by S. T. Yau [17] for S^2 in S^4 and by Kenmatsu [10] under the stronger assumption of a bound on $\langle N, A\rangle$. Lawson and Osserman [12] constructed a series of examples of

(1) Supported in part by NSF grant MGS 77-18723 A01.