

PERIODIC MINIMAL SURFACES AND WEYL GROUPS

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§ 1. Introduction

The observation that the Jacobi map of a compact Riemann surface X is universal among all harmonic maps of X into real tori is the basis for our investigation of periodic minimal surfaces in Euclidean space [11], [12] and [14]. This paper continues this work; some of the results were announced at the U.S.-Japan Seminar on Minimal Surfaces in 1977 [13].

Roughly, the first half extends our work [14] to minimal surfaces with symmetry in arbitrary codimension. The main result is that to any such conformal minimal immersion of a fixed compact Riemann surface in flat n -tori there corresponds a certain complex subvariety of its Jacobi variety and this correspondence is essentially unique (Theorem 3). An essential step in the proof, but also of intrinsic interest, is the observation that the image of any such immersion is homologous to zero (Theorem 2).

In the second half we develop an idea going back to the H. A. Schwarz Preisschrift [18] of 1867 to construct a remarkable family of such surfaces. We begin by solving a geometric problem of Schoenflies [17] in n dimensions; the solution shows how the root

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