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On Rebelo's theorem on singularities of holomorphic flows

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Introduction

A holomorphic vector field Z on a complex manifold M is said to be $\mathbf{R}^+$ complete, resp. $\mathbf{R}$ complete or $\mathbf{C}$ complete if the initial value problem

$$\phi(0)=p, \quad \phi'(t)=Z(\phi(t))$$

can be solved in forward time, $t>0$, resp. in real time, $-\infty<t<+\infty$, or in complex time, $t\in\mathbf{C}$. Of course, complete in complex time implies complete in real time implies complete in positive time. On any Stein manifold that does not support any bounded, non-constant, plurisubharmonic function, complete in positive time implies complete in complex time ([1], generalizing [9]). In some sense, fields complete in positive time are much more abundant than those complete in real time. For example, in the unit disk, among non-constant fields vanishing at the origin, only the rotation fields are complete in real time but any small perturbation of the field $Z(\zeta)=-\zeta$ is complete in positive time. Rebelo's theorem says the following.

Theorem. (Rebelo [13].) *If a* $\mathbf{C}$ *complete holomorphic vector field on a two dimensional complex manifold has an isolated zero at some point* p*, then at this point the two jet of the field is not zero.*

Our goal is to show that there are several ways to easily strengthen this result. We will use the following notation: if Z is a holomorphic vector field defined near a point p in some complex manifold, $J_k(Z,p)$ will denote the k jet of Z at p.

Proposition 1. *Let* M *be a complex manifold of dimension two. Let* Z *be an* $\mathbf{R}^+$ *complete holomorphic vector field on* M*. Assume that* Z *has an isolated zero at* p*. Then* $J_2(Z,p)\neq 0$*. If* $J_1(Z,p)=0$ *then there is an embedded Riemann sphere* Σ *in* M *such that* $p\in\Sigma$ *and* Z *is tangent to* Σ*.*