ON TWO THEOREMS OF F. CARLSON AND S. WIGERT.

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1. In this short note I have united a number of remarks relating to two theorems due in part to WIGERT and in part to CARLSON.[1] The theorems belong to the same region of the theory of functions, and it is natural to consider them together.

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2. I write $z = x + iy = re^{i\theta}$. Then the first theorem is as follows.[2]

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[1] The manuscript of the note (then entitled 'On two theorems of Mr. S. Wigert') was sent to Prof. MITTAG-LEFFLER in 1917. I was at that time unaware of the existence of Mr. CARLSON's dissertation ('Sur une classe de séries de Taylor', Uppsala, 1914). This dissertation was given to me by Prof. MITTAG-LEFFLER in September 1919; and I found at once that Mr. CARLSON had anticipated not only Mr. WIGERT's theorem of 1916, referred to in § 2, but my own generalisation of this theorem and indeed the substance of most that I had to say.

The note, however, contains something in substance, and a good deal in presentation, that is new; and I have therefore agreed to Prof. MITTAG-LEFFLER's suggestion that it should still appear. Except as regards §§ 1—2, I have left it substantially in its original form.

[2] WIGERT ('Sur un théorème concernant les fonctions entières', *Arkiv för Matematik*, vol. 11, 1916, no. 22, pp. 1—5) proves a theorem which is less general in that (1) the angle is supposed to cover the whole plane and (2) $f(z)$ is supposed to vanish for all positive and negative integral values of z. CARLSON (*l. c.*, p. 58) proves a theorem which contains the present theorem as a particular case (but is in fact substantially equivalent to it). His method of proof is similar to that of the first two proofs given here.

WIGERT (*l. c.*) refers to previous and only partially succesful attempts to prove his theorem, and gives a proof based on a theorem of PHRAGMÉN ('Sur une extension d'un théorème classique de la théorie des fonctions', *Acta Mathematica*, vol. 28, 1904, pp. 351—369). He deduces as a corollary a result relating to the case in which $f(z)$ vanishes only for positive integral values of z; in this the number π is replaced by the less favourable number $\frac{1}{2}\pi$. I may add that a similar result, in which $\frac{1}{2}\pi$ is replaced by the still less favourable number 1, was found independently by PÓLYA ('Über ganzwertige ganze Funktionen', *Rendiconti del Circolo Matematico*