THE CLASSIFICATION OF SETS OF POINTS.

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Preface.

The origin of this paper was a desire for a definition of a plane curve which should require a curve to be in some sense in one piece without requiring it to be closed or to be of the very special character of a Jordan curve. To take a simple example, there must be some sense in which a lemniscate deprived of any point but the node is a single curve but a lemniscate deprived of the node is two curves. The discovery of the property of a set which I define in section 20 and describe by saying that a set is united led to the definition of a curve on a surface which is given below in section 38 of the paper, but aroused a fresh discontent, since this definition gave no clue to the distinction in three-dimensional space between a curve and a surface, a distinction which the definition since evolved, given in section 36, enables me to draw.

Although the work was begun for the sake of a theory of dimensions, it is not on account of the theory suggested in the concluding sections that this paper is published; much remains to be done before that theory can be proved valuable or valueless. But of the thirty-nine sections of this paper the first thirty-five are concerned only with ideas which certainly have technical use as well as intrinsic interest; among these, the fundamental idea of which I have found 2 no

¹ These definitions of a united set and of a curve on a surface were given in a short note entitled »Definition of a plane curve» in *The Journal of the Indian Mathematical Society*, Vol. vii. pp. 175—177 (1915), the set being there described as perfectly connected.

² (Added November 1916) This statement only reveals my ignorance when it was written. Undeniable traces are to be found in Schoenflies' definition (*Math. Annalen*, bd. 58 (1904), s. 210) of a plane set of a special kind as coherent if every pair of its members can be joined by a simple path within the set, and in a footnote (*American Journal of Mathematics*, v. 33 (1911)