THE AUTOMORPHISMS AND THE ENDOMORPHISMS OF THE GROUP ALGEBRA OF THE UNIT CIRCLE

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1. Professor Boas recently suggested the following problem in a letter:
For what integer-valued sequences $\{t(n)\}$, $n=0, \pm 1, \pm 2, \ldots$, *is it true that*

$$\sum_{-\infty}^{\infty} a\,(t\,(n))\, e^{in\theta}$$

is a Fourier series whenever

$$\sum_{-\infty}^{\infty} a\,(n)\, e^{in\theta}$$

is a Fourier series?

Theorem I of the present paper contains the solution of this problem, and leads to a complete description of all automorphisms and endomorphisms of the group algebra of the unit circle, i.e., the algebra whose members are the Lebesgue integrable complex-valued functions on the unit circle, with convolution as multiplication. The algebra of all bounded complex Borel measures on the circle is also discussed from this standpoint.

Boas' question was prompted by the following theorem recently obtained by Leibenson [7] and Kahane [6] (the latter removed the differentiability conditions imposed on w by the former):

The only real functions w *which have the property that* $f\,(e^{i w(\theta)})$ *has an absolutely convergent Fourier series whenever the Fourier series of* $f\,(e^{i\theta})$ *converges absolutely, are of the form* $w\,(\theta)=n\,\theta+\alpha$, *where* n *is an integer and* α *a real number.*