A GENERAL PRIME NUMBER THEOREM.

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Consider a monotone sequence of real positive numbers

$$(1) \qquad 1 < y_1 < y_2 < \cdots < y_n < \cdots.$$

Form all possible products

$$(2) \qquad x = y_{n_1} y_{n_2} \ldots y_{n_k}, \qquad n_1 \leq n_2 \leq \cdots \leq n_k,$$

and arrange them in a non-decreasing sequence

$$(3) \qquad 1 < x_1 \leq x_2 \leq \cdots \leq x_n \leq \cdots$$

where every x appears as many times as it can be represented by formula (2). The numbers $\{y_n\}$ are called the primes of the sequence $\{x_n\}$. Let $\pi(x)$ denote the number of primes $\leq x$, and $N(x)$ the number of $x_n \leq x$.

This definition of generalized prime numbers is given by BEURLING, who under certain general conditions has derived very interesting relations between the functions $N(x)$ and $\pi(x)$.[1]

In what follows, $\zeta(s)$ denotes the function

$$(4) \qquad \zeta(s) = 1 + x_1^{-s} + x_2^{-s} + \cdots = \int_0^\infty x^{-s}\, d\, N(x). \qquad s = \sigma + it.$$

(For the sake of simplicity, we assume that $N(x)$ has a step equal to 1 at the point $x = 1$.) $\mathrm{Li}(x)$ denotes the logarithmic integral, i. e. the principal value of the integral

$$\int_0^x \frac{dy}{\log y}.$$

[1] A. BEURLING, Analyse de la loi asymptotique de la distribution des nombres premiers généralisés, Acta mathematica, vol. 68.