SPACES WITH NON-POSITIVE CURVATURE.

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Introduction.

The theory of spaces with negative curvature began with Hadamard's famous paper [9].[1] It initiated a number of important investigations, among which we mention Cartan's generalization to higher dimensions in [7, Note III], the work on symbolic dynamics[2] for which, besides Poincaré, Hadamard's paper is the ultimate source, and the investigations of Cohn-Vossen in [8], which apply many of Hadamard's methods to more general surfaces.

For Riemann spaces the analytic requirement that the space has non-positive curvature is equivalent to the geometric condition that every point of the space has a neighborhood U such that the side bc of a geodesic triangle abc in U is at least twice as long as the (shortest) geodesic arc connecting the mid points b', c' of the other two sides:

$$(*) \qquad bc \geq 2 \cdot b'c'.$$

This condition has a meaning in any metric space in which the geodesic connection is locally unique. *It is the purpose of the present paper to show, that* (*) *allows to establish the whole theory of spaces with non-positive curvature for spaces of such a general type.* This theory proves therefore *independent of any differentiability hypothesis and,* what is perhaps more surprising, *of the Riemannian character of the metric.*

It was quite impossible to carry all the known results over without swelling the present paper beyond all reasonable limits. But an attempt was made to

[1] The numbers refer to the References at the end of the paper.

[2] A bibliography is found in the paper [12] by M. MORSE and G. HEDLUND on this subject.