

CONCENTRATED AND RARIFIED SETS OF POINTS.

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CHAPTER I.

Concentrated Sets.

I have arrived at the definition of concentrated sets from the following two similar problems:

Problem I. *What are the linear sets whose measure with respect to any continuous monotone function $\varphi(l)$ is zero?*

Problem II. *What are the linear sets on which the variation of any continuous monotone function is zero?*

Concentrated sets are defined as follows:

A non-enumerable set of points E is said to be concentrated in the neighbourhood of an enumerable set H if any open set containing the set H contains also the set E with the exception of at most an enumerable set of points.

§ 1. **Measure with respect to a function.** Let $\varphi(l)$ be a positive increasing function satisfying defined for $l > 0$ and such that $\varphi(+0) = 0$ and let E be a linear set of points. Denote by $I = \Sigma l_i$ any sequence of intervals containing the whole of the set E (l_i denoting both an interval and its length). Denote by $m_\varphi^l E$ the lower bound of the sum

$$\sum \varphi(l_i)$$