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Asymptotic expansions of matrix coefficients of Whittaker vectors at irregular singularities

by

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0. Introduction

Singularities of systems of linear differential equations are usually classified into two classes: the regular type and the irregular type. When only one variable is involved, both types of singularities have been studied extensively in the literature. Some general tools have been developed, e.g., asymptotic expansions [Wa], and there are abundant families of examples, e.g., the confluent hypergeometric functions which include the classical Whittaker functions and Bessel functions [WW]. But no powerful general tools are available to handle irregular singularities in several variables.

An example is the system of differential equations satisfied by Whittaker functions on a semi-simple Lie group split over $\mathbf{R}$, which has irregular singularities at ∞ in every direction in the positive Weyl chamber. Since the Fourier coefficients of an automorphic form along the nilpotent radical of a parabolic subgroup are expressed in terms of Whittaker functions, a better understanding of their growth in every direction would be useful in the study of automorphic forms. In [MW], it was conjectured that the growth condition in the definition of automorphic form is superfluous for real semi-simple Lie groups with reduced real rank at least 2. In the same paper Miatello and Wallach [MW] have given a family of examples and one of the key steps in the estimates follows from the compactness of a certain set. This fails to be true in general, for example, $\mathrm{SL}(3,\mathbf{R})$. It seems that this failure may be compensated for by a better understanding of Whittaker functions. The present work is an initial probe to examine the phenomenon of irregular singularities through specific examples and a preparation for an understanding of the growth condition satisfied by automorphic forms.

The classical Whittaker functions have been studied in great detail in [WW]. In that reference, a convergent series expansion near 0 (on the negative chamber) and an asymptotic series expansion at ∞ (on the positive chamber) are given. Motivated by