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On the Dirichlet problem for Hessian equations

by

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1. Introduction

In this paper we consider the classical solvability of the Dirichlet problem for nonlinear, second-order elliptic partial differential equations of the form,

$$F(D^2u) \equiv f(\lambda[D^2u]) = \psi(x, u, Du), \tag{1.1}$$

in domains Ω in Euclidean n-space, $\mathbf{R}^n$, where f is a given symmetric function on $\mathbf{R}^n$, λ denotes the eigenvalues $\lambda_1, ..., \lambda_n$ of the Hessian matrix of second derivatives D^2u and ψ is a given function in $\Omega \times \mathbf{R} \times \mathbf{R}^n$. Equations of this type were treated by Caffarelli, Nirenberg and Spruck [2], for the case $\psi \equiv \psi(x)$, who demonstrated the existence of classical solutions for the Dirichlet problem, under various hypotheses on the function f and the domain Ω. Their results extended their previous work [1], and that of Krylov [13], Ivochkina [8] and others, on equations of Monge–Ampère type,

$$F(D^2u) = \det D^2u = \psi(x, u, Du). \tag{1.2}$$

Typical cases, embraced by [2] and treated as well by Ivochkina [9], are the elementary symmetric functions,

$$f(\lambda) = S_k(\lambda) = \sum_{i_1 < i_2 < ... < i_k} \lambda_{i_1} ... \lambda_{i_k}, \tag{1.3}$$

$k=1, ..., n$. Note that the case $k=1$ corresponds to Poisson's equation, while for $k=n$, we have the Monge–Ampère equation (1.2). If the function $\psi(x)$, boundary $\partial\Omega$ and boundary function ϕ are sufficiently smooth and ψ is uniformly positive in Ω, the classical Dirichlet problem,

$$\begin{aligned} F(D^2u) = S_k(\lambda[D^2u]) &= \psi \quad \text{in } \Omega, \\ u &= \phi \quad \text{on } \partial\Omega, \end{aligned} \tag{1.4}$$

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