A COMPLEX TENSOR CALCULUS FOR KÄHLER MANIFOLDS.

By

P. R. GARABEDIAN and D. C. SPENCER.

Introduction.

In this paper we develop a complex tensor calculus for Kähler manifolds and apply it to obtain results concerning analytic p-vectors on such manifolds. The Stokes and Brouwer operators d and δ are real operators in the sense that they send real p-vectors into real ones. We define complex analogues of these operators in terms of which the classical Laplace-Beltrami operator Δ for p-forms is split into a complex Laplace-Beltrami operator $\square$ and its conjugate $\bar{\square}$. In the case of scalars on Kähler manifolds and in the case of p-vectors of arbitrary degree p in Euclidean space we have $\square = \bar{\square} = \frac{1}{2}\Delta$.[1]

In Section 4 the complex operators are defined for currents of degree p (in the sense of de Rham). In Section 6 a method is given in terms of the complex operators for finding the complex-analytic projection of an arbitrary norm-finite p-vector, and in Section 7 this method is extended to currents. In Sections 8 and 9 the real operator Δ is investigated on submanifolds of the given manifold. Here the Kähler property of the metric is not used; therefore the results of Sections 8 and 9 are valid for Riemannian manifolds. In particular, it is established that every finite submanifold possesses a singular kernel $g_p(x, y)$ satisfying $\Delta_x g_p(x, y) = -\beta_p(y, x)$ where β_p is the reproducing kernel for harmonic p-forms (the existence of $g_p(x, y)$ on compact manifolds has been proved by de Rham).

[1] (Added in proof.) The complex operators as defined below were introduced by the authors in a report having the same title as this paper [Technical Report No. 17, Stanford University, California (May 21, 1951)]. The same operators, in a different notation, were introduced independently by Hodge [Proc. Cambridge Phil. Soc., 47 (July 1951)] who proved the equality of the operators $\square$ and $\bar{\square}$ in all cases. Since the present paper was submitted for publication before the appearance of Hodge's paper, we have not been able to use this identity to simplify some of the later portions of this paper. However, we remark that the identity $\square = \bar{\square}$ follows readily from formula (2.26) below and from Ricci's identity.