ONE-ONE MEASURABLE TRANSFORMATIONS.

By

CASPER GOFFMAN.

1. Introduction. The literature on the theory of functions of a real variable contains a variety of results which show that measurable functions, and even arbitrary functions, have certain continuity properties. As examples, I mention the well known theorems of Vitali-Carathéodory [1], Saks-Sierpinski [2], Lusin [3], and the theorem of Blumberg [4] which asserts that for every real function $f(x)$ defined on the closed interval $[0,1]$ there is a set D which is dense in the interval such that $f(x)$ is continuous on D relative to D.

The related topic of measurable and arbitrary one-one transformations has been given little attention. I know only of Rademacher's work [5] on measurability preserving transformations and my short paper [6] on the approximation of arbitrary one-one transformations.

My purpose here is to fill this void partially by obtaining for one-one measurable transformations an analog of Lusin's theorem on measurable functions. The form of Lusin's theorm I have in mind is that [7] for every measurable real function $f(x)$ defined on the closed interval $[0,1]$ there is, for every $\varepsilon > 0$, a continuous $g(x)$ defined on $[0,1]$ such that $f(x) = g(x)$ on a set of measure greater than $1 - \varepsilon$. The analogous statement for one-one transformations between $[0,1]$ and itself is that for every such one-one measurable $f(x)$ with measurable inverse $f^{-1}(x)$ there is, for every $\varepsilon > 0$, a homeomorphism $g(x)$ with inverse $g^{-1}(x)$ between $[0,1]$ and itself such that $f(x) = g(x)$ and $f^{-1}(x) = g^{-1}(x)$ on sets of measure greater than $1 - \varepsilon$. I shall show that this statement is false but that similar statements are true for one-one transformations between higher dimensional cubes.

I shall designate a one-one transformation by $(f(x), f^{-1}(y))$, where the functions $f(x)$ and $f^{-1}(y)$ are the direct and inverse functions of the transformation. I shall say that a one-one transformation $(f(x), f^{-1}(y))$ between n and m dimensional unit cubes I_n and I_m is measurable if the functions $f(x)$ and $f^{-1}(y)$ are both measurable,

17—533805. *Acta mathematica*. 89. Imprimé le 6 août 1953.