Acta Math., 168 (1992), 153–218

The sharp Markov property of the Brownian sheet and related processes

by

ROBERT C. DALANG[1] and JOHN B. WALSH

Tufts University
Medford, MA, U.S.A.

University of British Columbia
Vancouver, British Columbia, Canada

0. Introduction

The Brownian sheet $(W_t, t\in \mathbf{R}^2_+)$ has long been known to satisfy Paul Lévy's sharp Markov property with respect to all finite unions F of rectangles (see [W1, Ru]), meaning that

(0.1) $\mathcal{H}(F)$ and $\mathcal{H}(\bar{F}^c)$ are conditionally independent given $\mathcal{H}(\partial F)$,

where $\mathcal{H}(F)=\sigma(W_t, t\in F)$ represents the information one can obtain about the sheet by observing it only in the set F. However, (0.1) fails when F is the triangle $\{(t_1,t_2)\in \mathbf{R}^2_+: t_1+t_2<1\}$ [W1], leaving the impression that the sharp Markov property is valid only for a very restricted class of sets. In contrast, the weaker germ-field Markov property, in which one replaces $\mathcal{H}(\partial F)$ by the germ-field $\mathcal{H}^*(\partial F)=\bigcap \mathcal{H}(O)$ (where the intersection is over all open sets containing ∂F), is valid for all open sets in the plane [Ro, Nu]).

One natural explanation for this is the following: in the one-parameter setting, the Markov property of the solution of a stochastic differential equation is closely connected with uniqueness for the initial value problem. Something similar should be true in the plane. Now the Brownian sheet is the solution of a certain hyperbolic partial differential equation [W3], and its Markov property is closely connected to the uniqueness problem for the hyperbolic partial differential equation $\partial^2 u/\partial x\partial y=0$. It is well-known that the boundary data needed to pose the Cauchy problem for this equation are the values of the function on the boundary together with the normal derivative at non-

[1] The results presented here were obtained while this author was a visitor in the Department of Statistics, University of California at Berkeley and was supported by a National Science Foundation Postdoctoral Fellowship DMS-8807256.