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Behavior of the Bergman projection on the Diederich–Fornæss worm

by

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§ 1. Introduction

In this paper we show that the Bergman projection operator for certain smooth bounded pseudoconvex domains does not preserve smoothness as measured by Sobolev norms.

Let Ω be a smooth bounded pseudoconvex domain in $\mathbf{C}^n$ and let $P^{(t)}$ denote the orthogonal projection from $L^2(\Omega)$ onto the Bergman subspace $B(\Omega)=L^2(\Omega)\cap\mathcal{O}(\Omega)$ with respect to the weighted norm $\|f\|^{(t)}=(\int_D|f(z)|^2\,e^{-t\|z\|^2}dV)^{1/2}$. Let $W^k(\Omega)$ denote the Sobolev space consisting of functions whose derivatives of order $\leqslant k$ are in $L^2(\Omega)$. An important result of Kohn [Ko1] implies that $P^{(t)}$ maps $W^k(\Omega)$ to $W^k(\Omega)$ when $t\geqslant t_0(k,\Omega)$. On the other hand, there is a large collection of results implying that for certain types of domains the unweighted Bergman projection $P=P^{(0)}$ preserves W^k for all $k\geqslant 0$. (See [FK] for the strictly pseudoconvex case; for results on weakly pseudoconvex domains the reader may consult [Ko2], [Ca], [Si] and the recent [BSt1], [Ch] as well as the references cited therein. Most of these results are focused on the $\bar\partial$-Neumann operator rather than P; see [BSt2] for the connection. Also, except for [Ko1], positive results in this area are typically valid for any choice of smooth positive weight function on $\bar\Omega$.)

The question of whether or not P is similarly well-behaved for *all* weakly pseudoconvex domains has remained open for many years. In this paper we show that this is not the case; in fact when Ω is the so-called "worm domain" of Diederich and Fornæss then P does not map W^k to W^k when $k\geqslant\pi/$(total amount of winding). This latter quantity is explained in section 4 below, where the construction of the worm is reviewed and the main result is proved. The proof depends on computations for a piecewise Levi-flat model domain depending in turn on certain one-dimensional computations; these are treated in sections 3 and 2, respectively. Section 5 contains additional remarks and questions.