SUBSPACES AND QUOTIENTS OF $l_p \oplus l_2$ AND X_p(1)

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0. Introduction

Much progress has been made in recent years in describing the structure of $L_p = L_p[0, 1]$, and, in particular, the $\mathcal{L}_p$ spaces (complemented subspaces of L_p which are not Hilbert space) have been studied extensively. The obvious or natural $\mathcal{L}_p$ spaces are l_p, $l_p \oplus l_2$, $(l_2 \oplus l_2 \oplus \ldots)_p$ and L_p itself. These were the only known examples until H. P. Rosenthal [18] discovered the space X_p (see below). This space perhaps seemed pathological when first introduced; however, it now appears that X_p plays a fundamental role in the study of L_p and $\mathcal{L}_p$ spaces.

The discovery of X_p permitted the list of separable $\mathcal{L}_p$ spaces to be increased to 9 in number [18]. Then G. Schechtman [20], again using X_p, showed that there are an infinite number of mutually non-isomorphic separable $\mathcal{L}_p$ spaces, and recently Bourgain, Rosenthal and Schechtman [2] succeeded in constructing uncountably many such spaces. It now appears improbable that a complete classification of the separable $\mathcal{L}_p$ spaces will be obtained. However, it might be possible to classify the "smaller" $\mathcal{L}_p$ spaces. For example it was proved in [11] that the only $\mathcal{L}_p$ subspace of l_p $(1 < p < \infty)$ is l_p. Also all complemented subspaces of $l_p \oplus l_2$ and $(l_2 \oplus l_2 \oplus \ldots)_p$ are known (see [4], [21] and [17]). (X_p is, for $p > 2$, a $\mathcal{L}_p$ space which embeds into $l_p \oplus l_2$ and thus into $(l_2 \oplus l_2 \oplus \ldots)_p$, but does not embed into these spaces as a complemented subspace.)

One question with which we are concerned in this paper is "What are the $\mathcal{L}_p$ subspaces X of $l_p \oplus l_2$ $(1 < p \neq 2 < \infty)$?" We answer this in Section 2 for those X with an unconditional basis (although every separable $\mathcal{L}_p$ space is known to have a basis [10], it is a major un-

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