QUASICONFORMAL MAPPINGS AND EXTENDABILITY OF FUNCTIONS IN SOBOLEV SPACES

BY

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§ 1. Introduction

Let $\mathcal{D}$ be an open connected domain in $\mathbf{R}^n$, $n \geq 2$. If α is a multi-index, $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n) \in \mathbf{Z}_+^n$, the length of α, denoted by $|\alpha|$, is the integer $\sum_{j=1}^n \alpha_j$ and $D^\alpha = (\partial/\partial x_1)^{\alpha_1} \ldots (\partial/\partial x_n)^{\alpha_n}$. A locally integrable function f on $\mathcal{D}$ has a weak derivative of order α if there is a locally integrable function (denoted by $D^\alpha f$) such that

$$\int_{\mathcal{D}} f(D^\alpha \varphi)\, dx = (-1)^{|\alpha|} \int_{\mathcal{D}} (D^\alpha f)\, \varphi\, dx$$

for all C^∞ functions φ with compact support in $\mathcal{D}$. For $1 \leq p \leq \infty$, $k \in \mathbf{N}$, $L_k^p(\mathcal{D})$ is the Sobolev space of functions having weak derivatives of all orders α, $|\alpha| \leq k$, and satisfying

$$\|f\|_{L_k^p(\mathcal{D})} = \sum_{0 \leq |\alpha| \leq k} \|D^\alpha f\|_{L^p(\mathcal{D})} < +\infty.$$

An extension operator on $L_k^p(\mathcal{D})$ is a bounded linear operator

$$\Lambda\colon L_k^p(\mathcal{D}) \to L_k^p(\mathbf{R}^n) \equiv L_k^p$$

such that $\Lambda f|_{\mathcal{D}} = f$ for all $f \in L_k^p(\mathcal{D})$. We say that $\mathcal{D}$ is an extension domain for Sobolev spaces (E.D.S.) if whenever $1 \leq p \leq \infty$, $k \in \mathbf{N}$, there is an extension operator for $L_k^p(\mathcal{D})$.[2] The following theorem is by now well known.

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[2] We do not require Λ to be an extension operator also on $L_m^p(\mathcal{D})$ for $m < k$. In fact, the one which will be constructed does not have that property.