ON COMPACT KÄHLER MANIFOLDS OF NONNEGATIVE BISECTIONAL CURVATURE, II

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This paper is a sequel to the preceding paper [HSW]. This study of compact Kähler manifolds of nonnegative bisectional curvature was inspired by the recent solution of the Frankel conjecture by S. Mori ([M]) in a general algebraic setting, and subsequently by Siu and Yau ([SY]) in the special context of Kähler geometry. With the case of positive bisectional curvature out of the way, a general understanding of the case of nonnegative bisectional curvature is naturally the next order of business. For complex surfaces, the work of Howard and Smyth ([HS]) achieves a complete classification. In higher dimensions, the main conclusion of these two papers is that the study of compact Kähler manifolds of nonnegative bisectional curvature can be essentially reduced to the special case where simple connectivity and the isomorphism $H^2(M, \mathbf{Z}) \cong \mathbf{Z}$ are in addition assumed (the theorem of [HSW] and Theorem C below), and that with a mild positivity assumption these two desirable properties would follow in any case (Theorem E below). We begin by listing the main results; their proofs will be given in subsequent sections.

THEOREM A. *Let M be an n-dimensional compact Kähler manifold with nonnegative Ricci curvature. If the maximum rank of the Ricci tensor on M is $n-k$, then:*

(A) $h^{p,0}(M)=0$ *for $p=k+1, \ldots, n$ ($h^{p,q}(M)$ denotes the dimension of the space of harmonic (p, q)-forms).*

(B) $h^{1,0}(M) \leqslant k$, *and* $h^{1,0}(M)=0$ *iff $\pi_1(M)$ is finite.*

(C) *If in addition the bisectional curvature is nonnegative, then* $h^{1,0}(M)=k$.

For the next theorem, recall from [Wu2] that a covariant Hermitian tensor is *quasi-positive* iff it is positive definite at one point and positive semi-definite everywhere; the

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