MULTIPLE-POINT FORMULAS I: ITERATION

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An *r-fold point* of a map $f: X \to Y$ is a point x of X such that there exist $r-1$ other points of X and each has the same image under f as x. All r points must be "distinct", but some may lie "infinitely close" to others; that is, the infinitely close points determine tangent directions along the fiber $f^{-1}f(x)$. An *r-fold-point formula* is a polynomial expression in the invariants of f that gives, under appropriate hypotheses, the number of r-fold points or the class m_r of a natural positive cycle enumerating the r-fold points. One method for obtaining an r-fold-point formula is the method of iteration, the subject of this article. The setting will be algebraic geometry, but the method and the formulas have a universal character.

Multiple-point theory had its beginnings around 1850 and has attracted attention on and off ever since. About 1973 the field became highly active and has remained so. A survey is found in [10] Chapter V; it includes an introduction to the method of iteration, which at the time was beginning to blossom. Another survey, [11], concentrates on the results of this article and its sequel, [12].

The sequel will present another method for obtaining multiple-point formulas. Based on the Hilbert-scheme, it yields a deeper understanding of the theory and more refined formulas. The method also lends itself better to the study of an important special case, central projections.

The first general double-point formula was obtained in rational equivalence by Todd (1940); he derived it along with a residual-intersection formula, one from the other by induction on the dimension, but his reasoning is specious. Independently, Whitney (1941) gave a double-point formula for an immersion of differentiable manifolds. In [19] Ronga, inspired by Whitney, obtained the double-point formula in ordinary cohomology for a generic map with ramification in both the differential-geometric and complex-analytic

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(1) J. S. Guggenheim Fellow. This work was supported in part by NSF MCS-7906895.