Acta Math., 162 (1989), 143-199

On Giambelli's theorem on complete correlations

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1. Introduction

The point of departure for the following work was an attempt to prove a formula of Giambelli which gives explicit expressions for a large family of characteristic classes of complete correlations. This formula of Giambelli together with some related formulas of Schubert stand out in the rich flora of numbers obtained from enumerative geometric problems; they constitute a generel species, solving large classes of enumerative problems, and therefore are of particular interest.

We shall present below a proof of Giambelli's formula for complete correlations and also a similar formula for complete quadrics which we shall make more precise later in this introduction. A special case of Giambelli's formula for complete correlations is a beautiful formula of Schubert for the powers of the first characteristic class (see [S2]). It is interesting to note that in an earlier paper [S1], Schubert expressed the powers of the first characteristic class of complete quadrics in terms of a numeric function ψ_A for which he only had a recursive definition; comparing with the explicit formula obtained for correlations indicated that there should be an explicit formula for ψ_A: "Während aber die Ergebnisse der früheren Untersuchung *noch nicht studierte* aus Binomialcoefficienten zusammengesetzte Ausdrücke sind, so sind die Ergebnisse der neuen Untersuchung elegant gestaltete Determinanten der Binomialcoefficienten." We obtain the explicit formula requested by Schubert as a special case of our results.