WEIGHTED POLYNOMIAL APPROXIMATION ON ARITHMETIC PROGRESSIONS OF INTERVALS OR POINTS

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Introduction and definitions

The classical Bernstein problem on weighted polynomial approximation is as follows: Given a continuous function $W(x) \geqslant 1$ on $(-\infty, \infty)$ such that, for every $n \geqslant 0$,

$$\frac{|x|^n}{W(x)} \to 0 \quad \text{as} \quad x \to \pm\infty;$$

determine whether or not every continuous function $f(x)$ satisfying

$$\frac{f(x)}{W(x)} \to 0, \quad x \to \pm\infty$$

can be approximated uniformly by polynomials with respect to the weight W, that is, whether or not, corresponding to every such f, there exist polynomials P making

$$\sup_{-\infty<x<\infty} \frac{|f(x)-P(x)|}{W(x)}$$

arbitrarily small.

In this problem, whose solution is known, it is approximation over the whole real line that is in question. The present study is concerned with the similar problem that arises when the real line is replaced by certain unbounded subsets thereof, namely those obtained when a fixed segment is translated to and fro through all integral multiples of a fixed distance, or even by discrete subsets, like the set of integers.

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