

Estimates for the $\bar{\partial}$ -Neumann problem in pseudoconvex domains of finite type in \mathbb{C}^2

by

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§0. Introduction

The object of this paper is to construct a parametrix for the $\bar{\partial}$ -Neumann problem for arbitrary bounded pseudoconvex domains in \mathbb{C}^2 of finite type, and to use this parametrix to obtain sharp regularity results for the associated Neumann operator and for solutions of $\bar{\partial}u=f$. As an application, we obtain an extension of the Henkin–Skoda theorem, which characterizes the zero sets of functions in the Nevanlinna class in strictly pseudoconvex domains, to pseudoconvex domains of finite type in \mathbb{C}^2 .

The $\bar{\partial}$ -Neumann problem is a boundary value problem for an elliptic system of partial differential equations. Let $\Omega \subset \mathbb{C}^n$ be a smoothly bounded domain. Let U be a neighborhood of the boundary $\partial\Omega$ and let $\varrho: U \rightarrow \mathbb{R}$ be a defining function so that

⁽¹⁾ All three authors are supported by grants from the National Science Foundation.