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The structure of the automorphism group of an injective factor and the cocycle conjugacy of discrete abelian group actions

by

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§ 0. Introduction

The purpose of this paper is to give a proof of Connes' announcement on approximately inner automorphisms and centrally trivial automorphisms of an injective factor of type III for the first time, and to provide a classification, up to cocycle conjugacy, of actions of a discrete abelian or finite group on the unique injective factor of type III_1, which completes the final step of classification of actions of such groups on injective factors.

The study of automorphism groups has been a powerful method for understanding the structure of von Neumann algebras. Connes magnificently developed this approach in [4, 6, 7, 8]. Jones [15] and Ocneanu [18] followed the line of Connes [4, 6] and completed the classification of discrete amenable group actions on the unique approximately finite dimensional (AFD) factor of type II_1. Their work also provides useful tools for the case of type III. Sutherland–Takesaki [20] gave a classification of discrete amenable group actions on AFD factors of type III_λ, $0\leq\lambda<1$. Through their and Ocneanu's work, importance of two special classes of automorphisms became clear. The classes are the approximately inner automorphisms $\overline{\mathrm{Int}}(\mathcal{M})$ and the centrally trivial automorphisms $\mathrm{Cnt}(\mathcal{M})$ of a factor $\mathcal{M}$. Connes [5] announced a characterization of these classes for AFD factors of type III, but the proof has been unavailable for more than ten years since then, though this result was used in Lemma 2(a) of Connes [8], which together with Haagerup [13] established the uniqueness of AFD factors of type III_1, and also in the above-mentioned paper [20]. The characterization, announced in Connes [5, section 3.8] without proof, is as follows. (See [11] and [4] for notations.)