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Homology of Euclidean groups of motions made discrete and Euclidean scissors congruences

by

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The present work continues our investigations on the Scissors Congruence Problems. These investigations originated with the Third Problem of Hilbert that dealt with the scissors congruence problem in Euclidean 3-space. As indicated in [1], [5], [16], the non-Euclidean versions are just as interesting. They have an intimate connection with the Eilenberg–MacLane homology of certain classical Lie groups (namely, the isometry groups of the appropriate classical geometries) and come into contact with algebraic K-theory, Cheeger–Chern–Simons characteristic classes, as well as other topics. In all three series of classical geometries, the spherical version enters because the basic Dehn invarients require an understanding of the spherical scissors congruence problem. In a number of recent works, we have concentrated our efforts on the non-Euclidean cases, see [6], [18] for results and summaries in these directions. In spite of our efforts, the most complete results remain to be the theorems of Dehn–Sydler–Jessen showing that volume and Dehn invariants form a complete system of invariants for the scissors congruence problem in Euclidean spaces of dimensions 3 and 4. The original work of Sydler [19] was an incredible tour de force geometric arument in Euclidean 3-space. It was rapidly simplified by Jessen in [9] and extended to Euclidean 4-space. The simplification by Jessen employed techniques from homological algebra. Nevertheless, two of the geometric arguments of Sydler were retained in Jessen's work. The present work continues in the direction of the general theme that the scissors congruence problems should be formulated and solved in terms of the Eilenberg–MacLane homology of classical groups (with appropriate coefficients). The principal goal in the present

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