

Dynamics of quadratic polynomials, I–II

by

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1. Introduction

Rigidity is a fundamental phenomenon in hyperbolic geometry and holomorphic dynamics. Its meaning is that the metric properties of certain manifolds or dynamical systems are determined by their combinatorics. Celebrated works of Mostow, Thurston, Sullivan, Yoccoz, among others, provide us with examples of rigid objects. Moreover, this phenomenon is intimately linked to the universality phenomenon, to basic measure-theoretical and topological properties of systems, to the problem of describing typical systems.

In the setup of holomorphic dynamics the general rigidity problem can be posed as follows: