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Values of Brownian intersection exponents, II: Plane exponents

by

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1. Introduction

This paper is the follow-up of the paper [27], in which we derived the exact value of intersection exponents between Brownian motions in a half-plane. In the present paper, we will derive the value of intersection exponents between planar Brownian motions (or simple random walks) in the whole plane.

This problem is very closely related to the more general question of the existence and value of critical exponents for a wide class of two-dimensional systems from statistical physics, including percolation, self-avoiding walks and other random processes. Theoretical physics predicts that these systems behave in a conformally invariant way in the scaling limit, and uses this fact to predict certain critical exponents associated to these systems. We refer to [27] for a more detailed account on this link and for more references on this subject.

Let us now briefly describe some of the results that we shall derive in the present paper. Suppose that $B^1, ..., B^n$ are $n\geqslant 2$ independent planar Brownian motions started from n different points in the plane. Then it is easy to see (using a subadditivity argument) that there exists a constant ζ_n such that

$$\mathbf{P}[\forall i\neq j\in\{1,...,n\},\, B^i[0,t]\cap B^j[0,t]=\varnothing]=t^{-\zeta_n+o(1)} \tag{1.1}$$

when $t\to\infty$. We shall prove

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