ANALYTIC AND QUASI-INVARIANT MEASURES

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1. Let the real line R act as a topological transformation group on the locally compact Hausdorff space S. This means that we are given a group homomorphism $t \to T_t$ from the Abelian group R into the group of homeomorphisms of the topological space S with the property that the function $(t, p) \to T_t p$ from $R \times S$ to S is continuous. The action of R on S can be used to define the convolution of a measure on S with a function on R. Let $M(S)$ be the Banach space of bounded complex Baire measures on S and let $L^1(R)$ be the group algebra of R. The convolution of λ in $M(S)$ with f in $L^1(R)$ is the measure $\lambda * f$ in $M(S)$ given by

$$(\lambda * f)\, E = \int_R \lambda(T_{-t} E)\, f(t)\, dt$$

for all Baire subsets E of S. Convolution in turn can be used to associate with a measure on S a closed subset of R called the spectrum of the measure. Let λ be in $M(S)$ and let $J(\lambda)$ be the collection of all f in $L^1(R)$ with

$$\lambda * f = 0.$$

$J(\lambda)$ is a closed ideal in $L^1(R)$. The spectrum of λ, denoted by $\operatorname{sp}(\lambda)$, is the closed subset of R where all Fourier transforms of functions in $J(\lambda)$ vanish (i.e. $\operatorname{sp}(\lambda)$ is the hull of the ideal $J(\lambda)$). λ will be called analytic if $\operatorname{sp}(\lambda)$ is contained in the nonnegative reals, and λ will be called quasi-invariant if the collection of λ null sets is carried onto itself by the action of R on S. Thus to say that λ is quasi-invariant means that

$$|\lambda| (T_t E) = 0$$

for all t in R whenever E is a Baire subset of S with

$$|\lambda| E = 0,$$

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