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On inhomogeneous minima of indefinite binary quadratic forms

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1. Introduction

Let R_m denote the set of points of the ξ, η-plane defined by $-1 \leqslant \xi\eta \leqslant m$. An inhomogeneous lattice $\mathcal{L}$ is a set of points

$$\xi = \alpha x + \beta y, \quad \eta = \gamma x + \delta y \tag{1.1}$$

where (x, y) run through all numbers congruent (modulo 1) to any given numbers (x_0, y_0) respectively. $\Delta = \Delta(\mathcal{L}) = |\alpha\delta - \beta\gamma|$ is the determinant of $\mathcal{L}$. $\mathcal{L}$ is called admissible for R_m if it has no point in the interior of R_m. The critical determinant D_m of R_m is defined to be lower bound of $\Delta(\mathcal{L})$ over all admissible lattices $\mathcal{L}$. Barnes and Swinnerton-Dyer [1] have obtained the exact value of D_m for $21/11 \leqslant m \leqslant 2.1251 \ldots$. For $m \geqslant 3$, Blaney [2] has proved that

$$D_m \geqslant \sqrt{(m+1)(m+9)} \tag{1.2}$$

and equality holds for infinitely many values of m.

In this paper we shall obtain exact values of D_m for $3 \leqslant m \leqslant 3.9437 \ldots$ (Theorem 1). These results are better than those obtained by Blaney [2] and Dumir and Grover [3]. In Theorem 2, we shall obtain some lower bounds of D_m for $m \geqslant 4$ which are better than (1.2) above. For $m \in [3, 22/7]$, we find the first isolation i.e. if $\mathcal{L}$ is not equivalent to a special lattice $\mathcal{L}_0$, then $D_m \geqslant 4(9+7\sqrt{3})m/33$ (Theorem 3) and also observe that the second isolation is not possible for $3 \leqslant m \leqslant 22/7$. These results will be used by one of the authors in finding the successive minima of non homogeneous quadratic forms.

To obtain these results, we use the general theory of two dimensional inhomogeneous lattices developed in Barnes and Swinnerton-Dyer [1] (henceforth this paper will