ON FREE GROUPS AND THEIR AUTOMORPHISMS

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1. Introduction

The group A_n of automorphisms of a free group F_n on n free generators has been investigated by J. Nielsen [4]. Nielsen found generators and relations for A_n; it follows from his results that the elementary or t-transformations defined below generate A_n. Also, Nielsen found a recursive method to decide whether a given set of n elements of F_n generates the group. But for $n > 2$ it still remained an unsolved problem to decide whether a given element of F_n could appear in a set of free generators of F_n. This problem was solved by Whitehead [6]; in a subsequent paper, Whitehead [7] proved the following powerful theorem:

Given a set of words $W_0, \ldots, W_k$ in the generators of F_n, if the sum L of the lengths of these words can be diminished by applying automorphisms of F_n to the generators, then it can also be diminished by applying an automorphism of a pre-assigned finite set of automorphisms (the so-called T-transformations defined below).

The group A_n is of importance for Dehn's "isomorphism problem" of group theory (Dehn, [1]). Its most significant application is furnished by Grushko's theorem (see Kurosh [2] and B. H. Neumann [3]) which shows the following: given a minimal set of n generators of a group G which is a free product of a finite number of its subgroups H_q $(q = 1, \ldots, r)$. one can apply a transformation A of A_n to the generators a_j of G such that each of the resulting elements $A(a_j)$ belong to an H_q. The theorem of Whitehead and the theorem of Grushko have been used by Shenitzer [5] to devise tests for the free decomposability of groups with a single defining relation.

Whitehead uses difficult topological methods in proving his results. In the case where $n = 3$, a purely algebraic derivation of his theorems has been given by the