BAER-INVARIANTS, ISOLOGISM, VARIETAL LAWS AND HOMOLOGY

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Introduction

This paper is mainly concerned with the classification of groups and the independence of laws in varieties of groups. However, the basic ideas go over to other varieties in the sense of universal algebra, especially varieties of associative and Lie algebras. We also deal with various problems outlined below whose proper context is the theory of triple homology. In view of the somewhat unusual mixture of disciplines, we have been at pains to make as few demands on the reader as possible; in particular, for most of the paper, we do not assume any knowledge of homology.

Central to the whole paper are the groups $\mathfrak{V}M(G)$ and $\mathfrak{V}P(G)$, defined in § I.1 for any variety of groups $\mathfrak{V}$ and any group G. These are the Baer-invariants; the first modern treatment is due to Fröhlich [10], who considered associative algebras, and named the invariants after Baer's group-theoretical papers [2]. Further work on the Baer-invariants of associative algebras appears in Lue [26] and [27]. For a recent discussion that reverts to group theory and is more in the spirit of Baer's paper, see J. L. MacDonald [28]. The group $\mathfrak{V}M(G)$ is always abelian, and is the Schur multiplier of G if $\mathfrak{V}$ is the variety of abelian groups; $\mathfrak{V}P(G)$ is a central extension of $\mathfrak{V}M(G)$ by the verbal subgroup of G, and so coincides with $\mathfrak{V}M(G)$ if $G \in \mathfrak{V}$. In § I.2 we consider the classification of groups into $\mathfrak{V}$-isologism classes after P. Hall. The larger the variety $\mathfrak{V}$ the cruder the classification, all groups in $\mathfrak{V}$ falling into the same class. The problem of constructing the groups in an isologism class is postponed to § II.3, and the reader who wishes to get to this quickly should skip §§ I.3–4. In § I.3 we show how a slightly stronger property than independence of the laws of a variety $\mathfrak{V}$ can be dealt with in terms of $\mathfrak{V}P(G)$ for suitable G, and we calculate $\mathfrak{V}P(G)$ in certain cases. In § I.4 the non-finitely based variety of Vaughan-Lee's in [38] is used to construct non-finitely generated groups $\mathfrak{V}P(G)$. The calculations are rather involved, and the results are not used except to construct a counter-example in § II.4.

If G is a group in the variety $\mathfrak{V}$, and A and B are left and right $\mathfrak{V}G$-modules respectively, where $\mathfrak{V}G$ is a certain quotient ring of $\mathbf{Z}G$, various well known theories, which here coincide,